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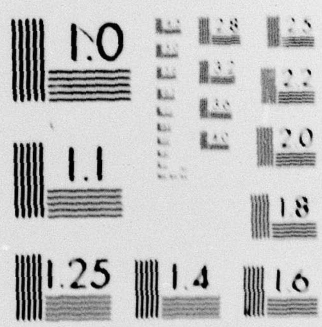

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GENERALIZATIONS OF THE SEQUENTIAL SIGN DETECTOR

C.C. LEE AND J.B. THOMAS
Department of Electrical Engineering and Computer Science
Princeton University, Princeton, New Jersey 08544

ABSTRACT

Some generalizations of the classical ruin problem in probability theory are investigated. These generalizations are applied to analyze the sequential dead-zone limiter detector and the sequential four-level sign detector. The performances of these detectors are then compared to that of the sequential sign detector in terms of the relative efficiency (RE) and the asymptotic relative efficiency (ARE).

I. INTRODUCTION

Sequential detection theory is known to be closely related to the well-known problem of random walk with absorbing barriers. As an example, the analogy between the sequential sign detector and the classical ruin problem will be illustrated. The advantages of a sign detector over the Neyman-Pearson optimal detector are its simplicity and nonparametric property. However, the hard-limiting nonlinearity of a sign detector often results in poor performance. A logical way to improve this is to increase the number of levels of quantization, while maintaining this number sufficiently small to retain the property of simple implementation. Thus, the "sequential dead-zone limiter detector", originally proposed by Shin and Kassam [3], and the "sequential 4-level sign detector" will be studied.

The counterpart of the sequential dead-zone limiter detector in random walk theory is a generalization of the ruin problem - "games with ties". By formulating the associated difference equations and solving them, or by simply using the conditional test and the available results from the usual ruin problem, we obtain expressions for the probabilities of false alarm and miss (probabilities of ruins under certain conditions) and for the average sample number (the expected duration of the game) required for pre-specified probabilities of errors.

To study the sequential four-level sign detector, we will consider another generalization of the ruin problem - "The stakes may be either ℓ_1 or ℓ_2 dollars instead of one dollar".
Now, except for some extremely special cases which are not of much interest to us, the associated difference equations cannot be solved analytically. Fortunately, the characteristic equation for the probability of ruin has exactly two positive roots (one of them is unity) so that we can find these and then use approximation techniques to solve numerically the difference equation for the probability of ruin (if the thresholds of the test are known). Also, when the probability of ruin (probability of false alarm or of miss) is pre-specified, we can solve for the thresholds needed. We then state a theorem by which the solution of the difference equation for the expected duration of the game (ASN) can be obtained or approximated with high accuracy without solving the equation.

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Generalizations to more-than-four-level input-quantization sequential detectors are straightforward. The complexity is not increased appreciably with the number of levels of quantization, as may be shown by formulating the difference equations and the procedures for analysis. However, as the number of levels increases, the advantage of simple implementation disappears.

To illustrate the performances of the sequential detectors discussed in this paper, we compare the expected sample numbers given the same error probabilities. The ratio of these numbers gives the relative efficiencies (RE), and the limit of RE as the signal strength vanishes gives the asymptotic relative efficiency (ARE). We derive an expression for the ARE of the sequential dead-zone limiter detector with respect to the sequential sign detector by assuming a symmetric noise distribution. Finally, by considering the generalized Gaussian noises, we compare the performances of the three detectors in terms of the RE. The breakpoint in the sequential dead-zone limiter detector and the sequential four-level sign detector, and the quantization levels in the latter, are chosen so as to give optimal performance. As may be expected, the performance improves as the number of quantization levels increases.

To introduce the sign detector, we treat the detection of constant antipodal signals in additive white noise:

$$H: X_i = N_i - s$$

vs.

$$K: X_i = N_i + s$$

where $s>0$ is a constant signal, and the noises N_i are assumed to have a symmetric and continuous common density function f. The observations $X_1, X_2, \ldots$ are assumed to be independent.

II. THE CLASSICAL RUIN PROBLEM AND THE SEQUENTIAL SIGN DETECTOR

The sequential sign detector has been studied in detail in [2]. The test scheme for this detector is the following: At the n-th sample, form

$$T_n \triangleq \sum_{i=1}^{n} \operatorname{sgn}(X_i) \begin{cases} \geq \ m_2 & \Rightarrow K \\ \leq -m_1 & \Rightarrow H \\ \text{otherwise} & \Rightarrow \text{take one more sample} \end{cases}$$

Clearly, T_n is the sum of a sequence of +1's and -1's, and the test procedure will be executed until T_n either exceeds m_2 or is less than $-m_1$.

Now, consider the classical ruin problem: a gambler either wins one dollar or loses one dollar on each game with probability p and q respectively ($p+q=1$). Let his initial capital be m_1 dollars and suppose he will continue to play until his capital either has increased to m_1+m_2 dollars or has reduced to zero. In the former case, he wins, and in the later case, he ruins. Let T_n' denote the increment in the gambler's capital; we have the following scheme: After the n-th game,

$$T'_n = \sum_{i=1}^{n} u_i \begin{cases} \geq \ m_2 & \Rightarrow \text{wins} \\ \leq -m_1 & \Rightarrow \text{ruins} \\ \text{otherwise} & \Rightarrow \text{plays the next game} \end{cases}$$

where u_i is +1 with probability p or -1 with probability q.

This scheme and that of the sequential sign detector are identical if we let

$$p = \Pr\{u_i=+1\} = \Pr\{\operatorname{sgn}(X_i)=+1\} = \Pr\{X_i>0\}$$

and

$$q = \Pr\{u_i=-1\} = \Pr\{\operatorname{sgn}(X_i)=-1\} = \Pr\{X_i<0\}$$

Thus, the probability of false alarm α for the sequential sign detector corresponds to the probability of winning (1-probability of ruin) for the ruin problem with $p=\Pr\{X_i>0|H\}$, and the probability of a miss $(1-\beta)$ corresponds to the probability of ruin with $p=\Pr\{X_i>0|K\}$. In addition, the average sample number (ASN) needed for the sequential sign detector corresponds to the expected duration of the game.

III. THE RUIN PROBLEM WITH "TIES" AND THE SEQUENTIAL DEAD-ZONE LIMITER DETECTOR

In this section, we consider a generalization of the ruin problem - the bet may result in a "tie". The gambler now may either win one dollar or lose one dollar or win (and lose) nothing on each game with probabilities p',q', and r' respectively $(p'+q'+r'=1)$. Let R_{m1} denote the probability that the gambler with initial capital m_1 will ruin. Since, after the first game, the gambler's fortune is either m_1+1 or m_1 or m_1-1 dollars, we have

$$R_{m1} = p'R_{m1+1} + q'R_{m1-1} + r'R_{m1} \tag{4.1}$$

with boundary conditions $R_0=1$, $R_{m1+m2}=0$. The solution for this difference equation is not difficult [9]. The result is

$$R_{m1} = \left[\left(\frac{q'}{p'}\right)^{m_1+m_2} - \left(\frac{q'}{p'}\right)^{m_1}\right] \Big/ \left[\left(\frac{q'}{p'}\right)^{m_1+m_2} - 1\right] \tag{4.2}$$

Note that this result is the same as that of the usual ruin problem [1] with p replaced by p' and q by q'.

Next, we consider the duration D_{m1} of the game. To find an expression for D_{m1}, we may solve the associated difference equation:

$$D_{m1} = p'D_{m1+1} + q'D_{m1-1} + r'D_{m1} + 1 \tag{4.3}$$

with boundary conditions $D_0=0$ and $D_{m1+m2}=0$. The solution is given in [9]. Here we show another approach which is much simpler and will be useful for further applications in the next section. The following theorem is intuitively true, and its proof is given in [9].

<u>Theorem 1</u>: "The expected duration of the game is equal to the expected gain divided by the expected gain in a single trial".

A direct result follows:

<u>Corollary 1</u>: The solution of (4.3) is given by

$$D_{ml} = [R_{ml}(-m_1) + (1-R_{ml})m_2]/(p'-q') \tag{4.4}$$

This result is the same as that obtained in [9] if we replace R_{ml} by (4.2). Theorem 1 enables us to solve the whole problem by solving the difference equation associated with the probability of ruin.

In the sequential sign detector, we make use of a hard-limiter (Fig. 1) to classify the samples.

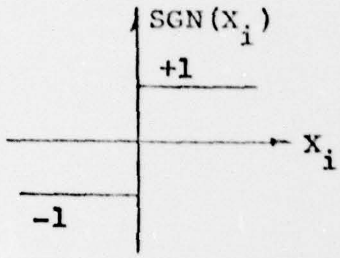

Fig. 1 A hard limiter

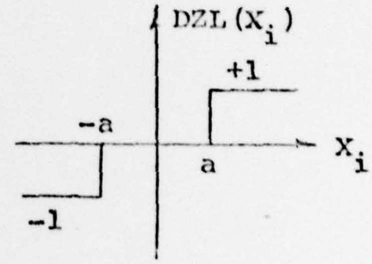

Fig. 2 A symmetric dead-zone limiter with dead-zone (-a,a)

In some cases, poor performance may result. If we use a dead-zone limiter (Fig. 2) instead, that is, if we eliminate a class of samples (those that lie in the dead-zone), the resulting detector may have significantly better performance characteristics. This sequential dead-zone limiter detector was originally proposed and analyzed, based on conditional tests, by Shin and Kassam [3]. We now use the ruin problem with ties to investigate this detector. The test scheme for a sequential dead-zone limiter detector is the following: At the n-th step

$$T_n = \sum_{i=1}^{n} DZL(X_i) \begin{cases} \geq \ m_2 & \Rightarrow K \\ \leq -m_1 & \Rightarrow H \\ \text{otherwise} & \Rightarrow \text{take one more sample} \end{cases}$$

where $DZL(X_i)$ has three possible values, +1,0, and -1. If we let

$$\begin{aligned} p' &= Pr\{DZL(X_i)=+1|H\} = Pr\{X_i>a|H\} = 1-F(a+s) \\ &= F(-a-s) = Pr\{X_i<-a|K\} = Pr\{DZL(X_i)=-1|K\} \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} q' &= Pr\{DZL(X_i)=-1|H\} = Pr\{X_i<-a|H\} = F(s-a) \\ &= 1-F(a-s) = Pr\{X_i>a|K\} = Pr\{DZL(X_i)=+1|K\} \end{aligned} \tag{4.6}$$

where F is the noise distribution function, the analogy between the ruin problem with ties and the sequential dead-zone limiter detector becomes apparent. It thus follows that

$$\begin{aligned} \alpha &= \text{Probability of win with parameters } p' \text{ and } q' \\ &= 1 - [(\tfrac{q'}{p'})^{m_1+m_2} - (\tfrac{q'}{p'})^{m_1}]/[(\tfrac{q'}{p'})^{m_1+m_2}-1] \end{aligned} \tag{4.7}$$

and

$1-\beta$ = Probability of ruin with parameters q' and p'

$$= [(\frac{p'}{q'})^{m_1+m_2} - (\frac{p'}{q'})^{m_1}]/[(\frac{p'}{q'})^{m_1+m_2}-1] \tag{4.8}$$

If α and $1-\beta$ are pre-specified, we may solve for m_1 and m_2:

$$m_1 = \ell og[(1-\beta)/(1-\alpha)]/\ell og(q'/p') \tag{4.9}$$

and

$$m_2 = \ell og(\alpha/\beta)/\ell og(q'/p') \tag{4.10}$$

Substituting these expressions into (4.4), we obtain the average sample number E(N). If H is true, R_{m1} is replaced by $1-\alpha$. If K is true, R_{m1} is replaced by $1-\beta$, p' by q' from (4.6), and q' by p' from (4.5).

IV. THE RUIN PROBLEM WITH "VARIABLE STAKES" AND n THE SEQUENTIAL 4-LEVEL SIGN DETECTOR

In this section, we investigate another generalization of the ruin problem - "the stakes may be either ℓ_1 or ℓ_2 ($\ell_2 > \ell_1$) dollars instead of one dollar". The gambler may either win ℓ_1 dollars, or win ℓ_2 dollars, or lose ℓ_1 dollars, or lose ℓ_2 dollars on each game with probabilities p_1, p_2, p_{-1}, p_{-2} respectively ($p_1+p_2+p_{-1}+p_{-2}=1$). Then, with initial capital m_1, the probability of ruin R_{m1} satisfies

$$R_{m1} = p_1 R_{m1+\ell 1} + p_2 R_{m1+\ell 2} + p_{-1} R_{m1-\ell 1} + p_{-2} R_{m1-\ell_2} \tag{5.1}$$

with boundary conditions

$$\begin{aligned} R_n &= 1 \quad , -\ell_2 + 1 \le n \le 0 \\ R_n &= 0 \quad , m_1 + m_2 \le n \le m_1 + m_2 + \ell_2 - 1 \end{aligned} \tag{5.2}$$

Unfortunately, this difference equation cannot be solved analytically except for some extremely special cases (e.g. $p_1=p_2=p_{-1}=p_{-2}=1/4$) which are not of great interest to us. But the situation is not too bad. If we substitute $R_m=x^m$ into the equation, we obtain its characteristic equation:

$$p_2 x^{2\ell_2} + p_1 x^{\ell_1+\ell_2} - x^{\ell_2} + p_{-1} x^{\ell_2-\ell_1} + p_{-2} = 0 \tag{5.3}$$

Clearly, unity is one of the roots of (5.3). Since the coefficients on the left side of (5.3) change signs twice, there is exactly one more positive root x_1. The x_1 can be located numerically [9] and then the solution of (5.1) can be bounded [9] as follows:

$$\frac{x_1^{m_1+m_2+\ell_2-1} - x_1^{m_1+\ell_2-1}}{x_1^{m_1+m_2+\ell_2-1}-1} \le R_{m1} \le \frac{x_1^{m_1+m_2+\ell_2-1} - x_1^{m_1}}{x_1^{m_1+m_2+\ell_2-1}-1} \tag{5.4}$$

Equation (5.4) can be further simplified to obtain, approximately

$$m_1 \le \ell ogR_{m1}/\ell ogx_1 \le m_1+\ell_2-1 \quad (5.5)$$

when m_1 and m_2 are large compared to ℓ_2 and the expected gain in a single trial is positive. The latter condition is what we are interested in when we study later the Sequential 4-Level Sign Detector. Since $m_1 >> \ell_2$, we may take $\ell ogR_{m1}/\ell ogx_1 \doteq (2m_1+\ell_2-1)/2$ or

$$R_{m1} \doteq \exp[(2m_1+\ell_2-1)\ell ogx_1/2] \quad (5.6)$$

Simulations show that this approximation is very accurate (to the fifth decimal when $m_1>100\ \ell_2$).

Once the probability of ruin R_{m1} is obtained, the following corollary of Theorem 1 in Section 4 is used to bound the expected duration D_{m1} of the game.

Corollary 2:

$$\frac{R_{m1}(-m_1-\ell_2+1)+(1-R_{m1})m_2}{p_1\ell_1+p_2\ell_2-p_{-1}\ell_1-p_{-2}\ell_2} \le D_{m1} \le \frac{R_{m1}(-m_1)+(1-R_{m1})(m_2+\ell_2-1)}{p_1\ell_1+p_2\ell_2-p_{-1}\ell_1-p_{-2}\ell_2}$$

It is apparent that the two bounds approach each other as m_1 and m_2 become very large compared to ℓ_2 as is usually the case in our applications.

As indicated in Sec. 1, to improve the poor performance caused in some cases by the hard-limiting nonlinearity of a sign detector, we may increase the levels of input-quantization. In [4], Kassam and Thomas proposed a 4-level sign detector which uses the nonlinearity shown in Fig. 3.

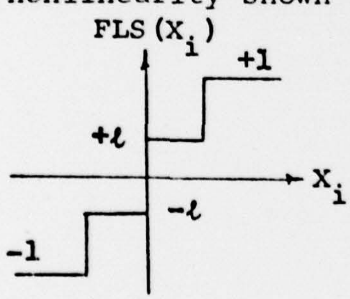

Fig. 3 Normalized 4-level quantizer

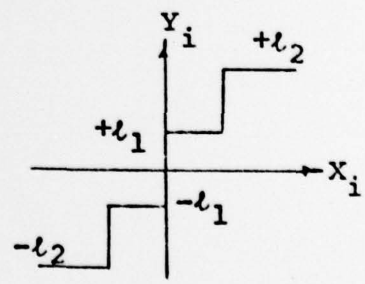

Fig. 4 4-level quantizer

We now investigate the sequential counterpart of this detector. We impose the following condition: $\ell=\ell_1/\ell_2$ is a rational number ($\ell_2>\ell_1$) so that the nonlinearity in Fig. 3 is equivalent to that in Fig. 4 if we properly adjust the test thresholds. Thus, the test scheme for a sequential 4-level sign detector is: at the *n*-th step,

$$T_n = \sum_{i=1}^{n} FLS(X_i) \begin{cases} \ge \ m_2 & \Rightarrow K \\ \le -m_1 & \Rightarrow H \\ \text{otherwise} & \Rightarrow \text{take one more sample.} \end{cases}$$

where FLS(X_i) has four possible values: $-\ell_2$, $-\ell_1$, $+\ell_1$ and $+\ell_2$. If we let

$$p_2=\Pr\{FLS(X_i)=+\ell_2|K\}=\Pr\{X_i>a|K\}=1-F(a-s)$$
$$=F(s-a)=\Pr\{X_i<-a|H\}=\Pr\{FLS(X_i)=-\ell_2|H\}$$

$$p_1=\Pr\{FLS(X_i)=+\ell_1|K\}=F(a-s)-F(-s)=F(s)-F(s-a)$$
$$=\Pr\{FLS(X_i)=-\ell_1|H\}$$

$$p_{-1}=\Pr\{FLS(X_i)=-\ell_1|K\}=F(-s)-F(-a-s)=F(s+a)-F(s)$$
$$=\Pr\{FLS(X_i)=+\ell_1|H\}$$

$$p_{-2}=\Pr\{FLS(X_i)=-\ell_2|K\}=F(-a-s)=1-F(a+s)=\Pr\{FLS(X_i)=+\ell_2|H\}$$

the analogy between this detector and the generalization of the ruin problem becomes obvious. We thus have

$$\alpha = \Pr\{\text{decide } K|H\} = \Pr\{\text{ruin}|\text{capital } m_2\} = R_{m2}$$

$$1-\beta = \Pr\{\text{decide } H|K\} = \Pr\{\text{win}|\text{capital } m_1\} = R_{m1}$$

For prespecified α and β, we may obtain m_1 and m_2 by using (5.5). Then, by applying Corollary 2, the average sample numbers (ASN) are obtained: $E(N|H) = D_{m2}$ and $E(N|K) = D_{m1}$.

V. PERFORMANCES

A. The Relative Efficiency (RE) and the Asymptotic Relative Efficiency (ARE) of the Sequential Dead-Zone Limiter Detector With Respect to the Sequential Sign Detector

The detection problem considered in this paper, as stated in Sec. 2, is "constant antipodal signals in additive white noise". Let f and F denote the density and the distribution function of the noise. By imposing only the condition that f is continuous and symmetric (with respect to the origin), we can derive [9] an expression for the RE of the sequential dead-zone limiter detector with respect to the sequential sign detector. The result is: under H and K.

$$RE21 = \frac{ASN_1}{ASN_2} = \frac{[1-2F(s+a)]\log\{F(s+a)/[1-F(s+a)]\}}{[1-2F(s)]\log\{F(s)/[1-F(s)]\}} \tag{6.1}$$

Certainly, we have to pick an optimal value for "a" to maximize $RE_{2,1}$. The generalized Gaussian density function [5]

$$f(x) = \frac{c\sigma^{-1}v(c)}{2\Gamma(1/c)} \exp[-(\sigma^{-1}v(c)|x|)^c] \tag{6.2}$$

represents a class of distributions with various rates of exponential decay, so it may be instructive to choose it as the noise distribution to check the performances of the detectors discussed in this paper. Tables 1 through 5 list RE21 for various values of c with α as a variable. The greater is c, the greater the RE21 that can be expected.

The ARE, defined as the limit of the RE as the signal strength goes to zero, is

$$ARE21 = f^2(a)/[2f^2(0)F(-a)] \qquad (6.3)$$

The derivation of (6.3) is also shown in [9]. Note that this equation is identical to that found in [3] and [4]. Fig. 5 shows this ARE for generalized Gaussian noise with "a" as a variable. We can see from the figure that, in the Gaussian case (c=2), the ARE may achieve a value greater than 1.27. Thus, the improvements made by the dead-zone limiter detector is significant.

B. Comparisons of the Sequential Dead-Zone Limiter Detector and The Sequential 4-Level Sign Detector with the Sequential Sign Detector in Terms of the RE

Analytic expressions for the RE's of the sequential 4-level sign detector (w.r.t. the sequential sign detector) are not available. Numerical results for this detector are obtained for the following three cases:

(i) $\ell_1=1$, $\ell_2=2$ (Detector #3)

(ii) $\ell_1=1$, $\ell_2=3$ (Detector #4)

(iii) $\ell_1=1$, $\ell_2=4$ (Detector #5)

Tables 1 through 5 give the RE's of Detector #2 through Detector #5 with respect to Detector #1 (the sign detector). Table 3 is for the Gaussian noise in which the RE for the corresponding sequential linear detector (Detector #6), which is Neyman-Pearson optimal for Gaussian noise, is also included for convenience of comparisons.

These tables show significant improvements over the sequential sign detector. The sequential 4-level sign detector performs better than the dead-zone limiter detector, as may be expected. As a matter of fact, the sequential dead-zone limiter detector is, of course, a degenerate sequential 4-level sign detector ($\ell_1=0$, $\ell_2=1$).

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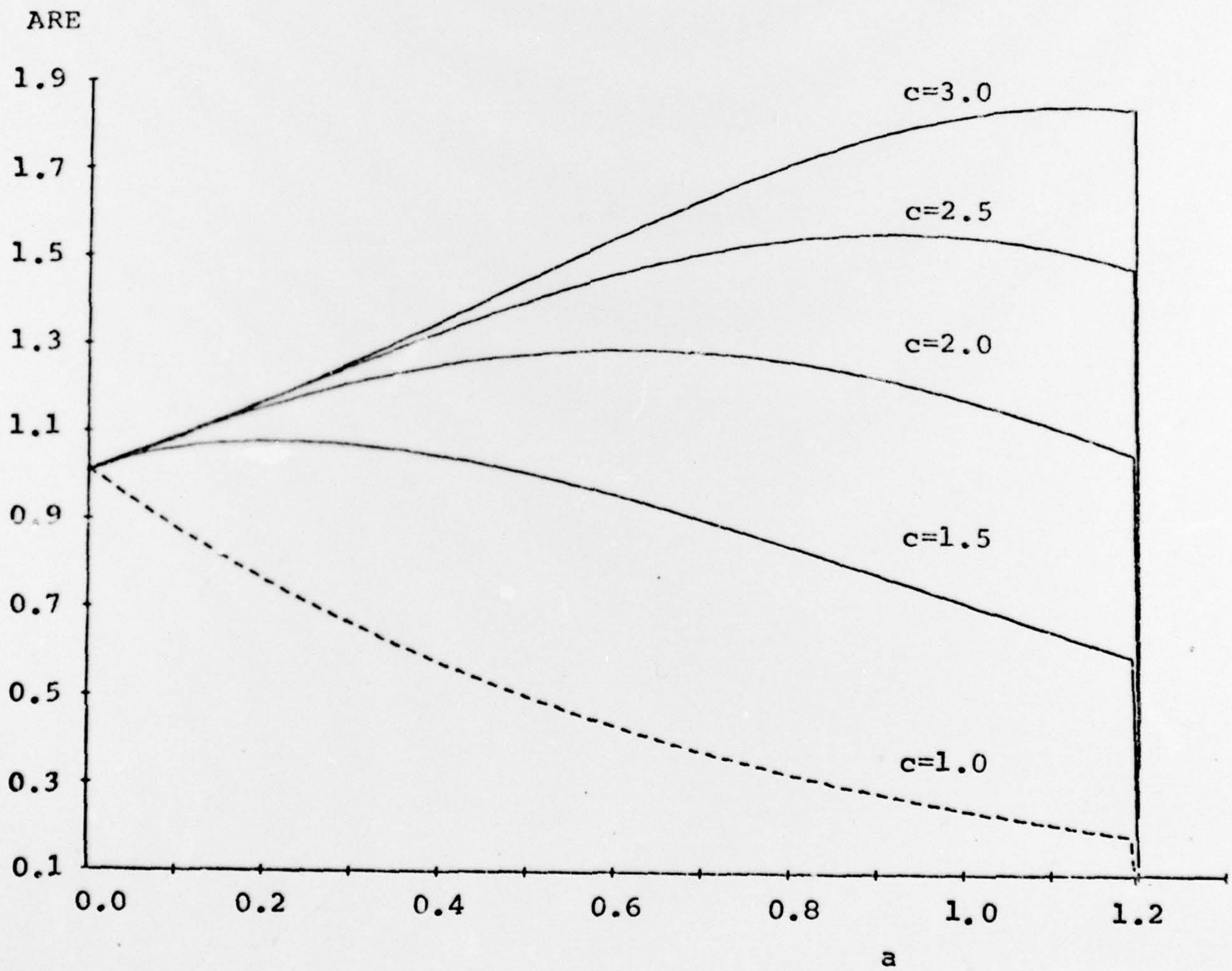

Fig. 5 The ARE of the sequential dead-zone limiter with respect to the sequential sign detector. Generalized Gaussian noises.

s/σ	0.1		0.01	
α	10^{-2}	10^{-6}	10^{-2}	10^{-6}
RE21	1.031	1.031	1.003	1.004
RE31	1.051	1.040	1.005	1.004
RE41	1.058	1.044	1.006	1.005
RE51	1.044	1.039	1.005	1.004

Table 1

c=1.0 (Laplace Noise)

s/σ	0.1		0.01	
α	10^{-2}	10^{-6}	10^{-2}	10^{-6}
RE21	1.080	1.080	1.065	1.065
RE31	1.162	1.151	1.141	1.139
RE41	1.159	1.145	1.130	1.128
RE51	1.131	1.126	1.113	1.113

Table 2

c=1.5

s/σ	0.1		0.01	
α	10^{-2}	10^{-6}	10^{-2}	10^{-6}
RE21	1.273	1.273	1.270	1.270
RE31	1.342	1.329	1.324	1.323
RE41	1.414	1.395	1.387	1.385
RE51	1.392	1.385	1.383	1.382
RE61	1.573	1.573	1.571	1.571

Table 3

c=2 (Gaussian Noise)

s/σ	0.1		0.01	
α	10^{-2}	10^{-6}	10^{-2}	10^{-6}
RE21	1.531	1.531	1.533	1.533
RE31	1.492	1.479	1.474	1.473
RE41	1.660	1.635	1.628	1.625
RE51	1.675	1.667	1.665	1.664

Table 4

c=2.5

s/σ	0.1		0.01	
α	10^{-2}	10^{-6}	10^{-2}	10^{-6}
RE21	1.815	1.815	1.818	1.818
RE31	1.619	1.604	1.600	1.598
RE41	1.881	1.852	1.843	1.840
RE51	1.944	1.934	1.931	1.930

Table 5

c=3.0